Properties of integrals

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1 Areas

$\int_a^b f(x)dx$ is the **signed area** under the curve of f

Problem 1[5.2.34] Evaluate (a) $\int_0^2 g(x)dx$, (b) $\int_2^6 g(x)dx$, (c) $\int_0^7 g(x)dx$, where the graph of g is given in the following picture (see lecture, or look up the picture at the very bottom of page 377)

Problem 2[5.2.36] Evaluate $\int_{-2}^2 \sqrt{4-x^2}dx$

2 Linearity properties

$$\int_a^a f(x)dx = 0, \qquad \int_b^a f(x)dx = -\int_a^b f(x)dx$$

$$\int_a^b f(x)+g(x)dx = \int_a^b f(x)dx+\int_a^b g(x)dx, \qquad \int_a^b f(x)-g(x)dx = \int_a^b f(x)dx-\int_a^b g(x)dx$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx, \qquad \int_a^b cdx = c(b-a)$$

Problem 3 Evaluate

$$\int_\pi^\pi e^{x^2}\sin(x)\left(\cos(x)^{e^{3x^3}} + \ln(\tan(2^x))\right)dx$$

Problem 4[5.2.42] If $\int_0^1 3x\sqrt{x^2+4}dx = 5\sqrt{5}-8$, what is $\int_0^1 3u\sqrt{u^2+4}du$?

3 Additivity Property

$$\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$$

Problem 5[5.2.47] Write $\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx$ as a signle integral

Problem 6 If $\int_1^5 f(x)dx = 10$ and $\int_4^5 f(x)dx = 4$, find $\int_1^4 f(x)dx$

4 Comparison Properties

$$\text{If} \quad f(x) \geq 0 \quad \text{then} \quad \int_a^b f(x)dx \geq 0$$

$$\text{If} \quad f(x) \leq g(x) \quad \text{then} \quad \int_a^b f(x)dx \leq \int_a^b g(x)dx$$

$$\text{If} \quad f(x) \leq g(x) \leq h(x) \quad \text{then} \quad \int_a^b f(x)dx \leq \int_a^b g(x)dx \leq h(x)$$

Problem 7[5.2.52] Show $\int_0^1 \sqrt{1+x^2}dx \leq \int_0^1 \sqrt{1+x}dx$

Problem 8[5.2.54] Show

$$\frac{\sqrt{2}\pi}{24} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos(x)dx \leq \frac{\sqrt{3}\pi}{24}$$